



# Balancing Efficiency and Risk in Public Sector Artificial Intelligence with Data Envelopment Analysis and Portfolio Approaches

Doroteja Mitrović<sup>1,\*</sup>, Gülay Demir<sup>2</sup>, Ibrahim Badi<sup>3</sup>, Mouhamed Bayane Bouraima<sup>4</sup>

<sup>1</sup> Faculty of Organizational Sciences, University of Belgrade, Belgrade, Serbia

<sup>2</sup> Sivas Cumhuriyet University, Vocational School of Health Services, Sivas, 58040, Türkiye

<sup>3</sup> Department Mechanical Engineering, Libyan Academy-Misrata, 2949 Misrata, Libya

<sup>4</sup> International School of Technical Education, Sichuan College of Architectural Technology, Deyang, 618000, Sichuan, China

## ARTICLE INFO

### Article history:

Received 25 May 2025

Received in revised form 31 July 2025

Accepted 15 September 2025

Available online 2 October 2025

### Keywords:

Artificial intelligence, AI readiness, Public sector efficiency, Fuzzy risk assessment, Resource optimization

## ABSTRACT

The integration of Artificial Intelligence (AI) in the public sector offers potential for improved efficiency but faces challenges related to readiness, infrastructure, and strategic planning. This study evaluates AI adoption across 151 countries using a novel framework that combines Data Envelopment Analysis (DEA), Fuzzy Logic, and Modern Portfolio Theory (MPT). DEA assesses efficiency levels, identifying top-performing nations and highlighting areas for improvement. Window analysis captures dynamic efficiency trends, providing a temporal perspective on AI readiness. Fuzzy Logic evaluates risks associated with AI implementation, focusing on key aspects such as governance, technology, and infrastructure. MPT optimizes resource allocation by balancing efficiency and risk, offering tailored investment strategies. The results identify Data Representativeness and Adaptability as critical dimensions for successful AI adoption, with significant disparities across nations. This approach provides actionable insights for policymakers to improve AI integration, reduce risks, and maximize benefits in public sector operations. The study underscores the need for dynamic, multi-dimensional strategies to address evolving challenges in AI implementation globally.

## 1. Introduction

The advancement of AI has led governments to consider its integration in the public sector to improve efficiency in service delivery. However, many lack the infrastructure, readiness, and strategic direction required for effective implementation. This study examines the challenges of AI adoption in the public sector across various countries, focusing on efficiency evaluation, risk assessment, and optimal resource allocation. Using Oxford's AI Readiness Index [1], the research assesses national preparedness, recognizing that successful AI integration requires both technological capability and skilled personnel.

The study applies DEA to measure efficiency levels in AI adoption across countries, establishing benchmarks for identifying well-prepared nations. Further, it uses Fuzzy Logic to evaluate the risks of

\* Corresponding author.

E-mail address: [dorotejaamitrovic@gmail.com](mailto:dorotejaamitrovic@gmail.com)

deploying AI in government contexts, where regulatory, operational, and public trust factors vary significantly. This risk assessment underscores the importance of strategic planning and ethical considerations. To guide resource allocation for improving efficiency, the study applies MPT to create an optimal investment portfolio for AI in public services, balancing efficiency and risk. Cross-country comparisons provide insights into best practices, supporting a strategic and informed approach to AI adoption. This research ultimately aims to help governments leverage AI effectively, reducing risks while improving public sector performance globally.

In existing literature, the role of AI in enhancing efficiency across sectors has been well-documented. Studies have explored its potential in management information systems, where AI is shown to improve predictive analytics, automate tasks, and support decision-making through case studies from various sectors [2]. The use of AI in public sector recruitment has been explored, showing that while it improves predictions of candidate suitability for specific roles, disclosing its use may discourage applicants [3]. These findings highlight both the opportunities and challenges associated with AI implementation in public administration.

DEA has been widely applied in the public sector for performance evaluation. For instance, an improved DEA algorithm has been used to address public management issues, providing better evaluation tools for public administration [4]. Similarly, DEA has been employed to measure the efficiency of e-governments in EU countries, focusing on digitization and public interaction with government systems [5]. Despite these applications, DEA has not been extensively used to address AI-specific challenges within the public sector.

Fuzzy Logic offers a complementary approach by addressing uncertainties inherent in decision-making processes. For example, it has been used to evaluate agility in public sector organizations under uncertain conditions [6] and to support investment-intensive decision-making by integrating diverse stakeholder perspectives [7]. However, its application in assessing AI-related risks and guiding public sector strategies remains limited.

Markowitz's MPT provides a framework for resource allocation by balancing risks and returns, primarily in financial contexts. While its use in public sector AI projects is unexplored, studies combining MPT with DEA and Fuzzy Logic have demonstrated its potential in portfolio selection under uncertainty. For instance, research using Markowitz's model and DEA cross-efficiency analysis has proven effective in selecting portfolios by accounting for market fluctuations with fuzzy numbers [8; 9]. These methodologies hold promises for managing resource allocation in AI adoption.

Despite these individual contributions, there is a significant gap in integrating DEA, Fuzzy Logic, and MPT into a unified framework for evaluating AI adoption in the public sector. Current research does not address efficiency, risk, and resource optimization simultaneously. This study bridges that gap by combining these methodologies to develop a strong model that supports governments in improving AI implementation, enabling informed and strategic decisions for effective public service delivery.

### *1.1 Aims of the Study*

The primary aim of this work is to identify countries that efficiently implement AI within the public sector, pinpointing where AI is effectively integrated and where improvements are needed. Building on these findings, the study then evaluates the risk associated with each country's AI implementation status to create optimal portfolios tailored to their efficiency and risk profiles. By linking efficiency with risk, the study provides actionable insights to improve AI adoption. Ultimately, the research aims to develop an optimal portfolio strategy for each country, balancing efficiency and risk to maximize AI benefits in public sector operations.

### *1.2 Motivations for Developing Methodology*

Existing approaches lack comprehensive analysis of AI implementation within the public sector. No studies to date evaluate AI's efficiency in this context or propose strategies for improvement. Current literature either focuses on general AI applications or specific industries, without assessing public sector effectiveness or unique risks. Furthermore, methodologies like DEA, Fuzzy Logic, and MPT are unused in measuring AI's impact on public sector efficiency, leaving significant gaps in understanding and strategic deployment.

This research fills significant gaps by presenting a targeted approach that evaluates AI implementation efficiency in the public sector across countries. Using DEA for efficiency assessment and integrating MPT and Fuzzy Logic to address risks, this study develops a framework responsive to varying national risk levels. It assesses current AI adoption effectiveness and develops a general portfolio framework that countries can apply based on their specific risk levels and efficiency profiles. This approach advances the understanding of AI's role in public sector efficiency and offers actionable insights for countries to improve AI deployment, balancing both effectiveness and risk.

### *1.3 Organization of the Study*

This study is organized into five chapters. Chapter One provides an introduction, outlining the research background, objectives, and significance of the study. Chapter Two reviews existing literature on methodologies used in analysis of AI implementation in the public sector, including DEA, Fuzzy Logic, and MPT. Chapter Three details how mentioned methodologies were employed in this research. Chapter Four presents the research findings, offering a comparative analysis of AI efficiency in public sectors across various countries. Chapter Five concludes the study by summarizing the key findings, discussing their implications, and suggesting areas for further research.

## **2. Methodology**

The methodology follows a structured approach to assess the efficiency and risk of AI implementation in the public sector. Initially, DEA is applied to determine the relative efficiency of different countries, identifying efficient and inefficient units based on AI adoption indicators. The analysis uses both CCR and Super-efficiency models, incorporating Window analysis to examine efficiency trends over time. DEA results establish benchmarks and highlight areas for improvement. Next, Fuzzy Logic is employed to assess the risk associated with AI implementation in each country, using aggregated data to capture the uncertainty in efficiency factors. This phase transforms quantitative data into linguistic variables, enabling a clearer understanding of risk dimensions across nations. Finally, MPT identifies optimal investment portfolios created to the efficiency and risk profiles of each country. The research flowchart is shown below (Figure 1).

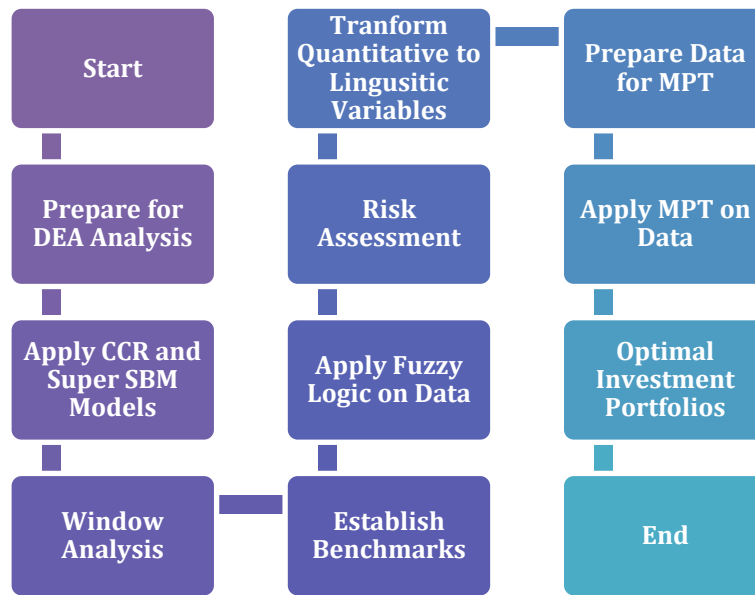


Fig. 1. Research flowchart

## 2.1 Efficiency Analysis Using Data Envelopment Analysis

DEA is a linear programming methodology that measures the relative efficiency of decision-making units (DMUs), which convert multiple inputs into multiple outputs [10]. It identifies the "efficient frontier," distinguishing between efficient and inefficient units based on resource use and output performance [11]. DEA helps identify best practices [12], including ones in public administration, enabling improved resource management and potential cost savings.

In this research, the Charnes, Cooper, & Rhodes (CCR) [13] model was initially applied to four-year average data, providing a clear efficiency comparison among countries. The CCR model assumes constant returns to scale (CRS), meaning that proportional changes in inputs yield equivalent changes in outputs, making it suitable for scenarios with stable scaling effects. This model identifies efficient units, or DMUs that serve as performance benchmarks for other units [10]. The primary output-oriented CCR DEA model, calculated as the reciprocal value of the input model, has the following form [14]:

$$\min g_k = \frac{\sum_{i=1}^m v_i x_{ik}}{\sum_{r=1}^s u_r y_{rk}} \quad (1)$$

Subject to:

$$\frac{\sum_{i=1}^m v_i x_{ij}}{\sum_{r=1}^s u_r y_{rj}} \geq 1, j = 1, \dots, n \quad (2)$$

$$u_r \geq 0, r = 1, \dots, s, v_i \geq 0, i = 1, \dots, m, u^0 \in \{-\infty, \infty\} \quad (3)$$

Where:

$g_k$ : Efficiency score for DMU under evaluation.

$x_{ik}$ : Input  $i$  for DMU  $k$ .

$y_{rk}$ : Output  $r$  for DMU  $k$ .

$u_r$ : Weight assigned to output  $r$ .

$v_i$ : Weight assigned to output  $i$ .

$n$ : Number of DMUs.

$s$ : Number of outputs.

$m$ : Number of inputs.

To address the limitations of the CCR model in differentiating between highly efficient units, Super-Slack-Based Measure (SBM) efficiency analysis was applied. This approach extends the CCR model by allowing DMUs to exceed an efficiency score of 1, enabling a more granular ranking of efficient units [14]. It distinguishes units that outperform others, highlighting those that significantly exceed average efficiency levels [15]. Below is shown objective function of Super-SBM analysis:

$$\delta^* = \min \frac{1 + \frac{1}{m} \sum_{i=1}^m \frac{s_i^-}{x_{i0}}}{1 - \frac{1}{m} \sum_{i=1}^s \frac{s_i^+}{y_{r0}}} \quad (4)$$

Subject to:

$$x_0 = \sum_{j=1}^n x_j \lambda_j - s_i^-, j \neq 0 \quad (5)$$

$$y_0 = \sum_{j=1}^n y_j \lambda_j + s_i^+, j \neq 0 \quad (6)$$

$$\lambda \geq 0, s_i^- \geq 0, s_i^+ \geq 0 \quad (7)$$

$$\sum_{j=1}^n \lambda_j = 1 \quad (8)$$

Where:

$\delta^*$ : Efficiency score for DMU under evaluation.

$x_{ik}$ : Input  $i$  for DMU  $k$ .

$y_{rk}$ : Output  $r$  for DMU  $k$ .

$\lambda_j$ : The weight for the  $j$ -th DMU in the linear combination.

$s^- = (s_1^-, s_2^-, \dots, s_m^-)^T \in R^m$ : Slack for the  $i$ -th input.

$s^+ = (s_1^+, s_2^+, \dots, s_m^+)^T \in R^S$ : Slack for the  $r$ -th output.

$n$ : Number of DMUs.

$s$ : Number of outputs.

$m$ : Number of inputs.

Additionally, Window analysis was employed to capture dynamic efficiency trends over time. This methodology uses a "moving window" approach, where the dataset is divided into overlapping time segments, enabling the assessment of efficiency variations across consecutive years [16]. It provides insights into temporal trends and helps identify factors that influence AI implementation success over the study period [13]. The window in time period  $l$  (ranging from 1 to  $P$ ) with width  $w$  (starting at  $t = l$  extending to  $l + w$ ) is denoted as  $l_w$  and contains  $n \times w$  DMUs. The input matrix  $X^{l_w}$  and the output matrix  $Y^{l_w}$  are represented as follows:

$$X^{l_w} = \begin{bmatrix} x_{11}^{(l)} & x_{12}^{(l)} & \dots & x_{1m}^{(l)} \\ x_{21}^{(l)} & x_{22}^{(l)} & \dots & x_{2m}^{(l)} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1}^{(l+w-1)} & x_{n2}^{(l+w-1)} & \dots & x_{nm}^{(l+w-1)} \end{bmatrix}; Y^{l_w} = \begin{bmatrix} y_{11}^{(l)} & y_{12}^{(l)} & \dots & y_{1s}^{(l)} \\ y_{21}^{(l)} & y_{22}^{(l)} & \dots & y_{2s}^{(l)} \\ \vdots & \vdots & \ddots & \vdots \\ y_{n1}^{(l+w-1)} & y_{n2}^{(l+w-1)} & \dots & y_{ns}^{(l+w-1)} \end{bmatrix}$$

Where:

$X^{l_w} / Y^{l_w}$ : Represents the input data for  $n \times w$  DMUs in the specified time window  $l_w$ .

$x_{ij}^{(t)} / y_{ij}^{(t)}$ : The  $i$ -th input for the  $j$ -th DMU at time period  $t$ .

$i$ : Index of the input type (e.g., resources, cost, etc.), ranging from 1 to  $m$  (total number of inputs).

$j$ : Index of the DMU being analyzed, ranging from 1 to  $n$  (total number of DMUs).

$t$ : Specific time period within the window  $l_w$  starting from  $l$  and ending at  $l+w-1$ .

$n$ : Number of DMUs.

$s$ : Number of outputs.

$m$ : Number of inputs.

$w$ : The width of the time window.

$l$ : The starting time period of the window.

DEA enables the evaluation of multiple inputs and outputs without requiring explicit definitions of functional relationships, making it a flexible tool for complex assessments in sectors with intricate input-output relationships [17]. However, DEA is sensitive to sample size and outliers, which can introduce biases in the presence of extreme values [18].

## 2.2 Risk Assessment with Fuzzy Logic

Fuzzy Logic is a multi-valued logic system where truth values range between 0 and 1, representing varying degrees of truth [19]. This allows it to represent partial truths rather than absolute true or false values, as seen in binary logic. Classical sets state rigidly, e.g., "x is between 3 and 5," while Fuzzy sets allow flexible statements like "x is close to 4," reflecting contextual factors [20]. Unlike binary logic, which is strictly true or false, fuzzy logic provides a gradient of truth values, making it suitable for handling uncertainty in complex scenarios [21]. The concept of linguistic variables, introduced by Lotfi Zadeh, extends fuzzy logic by using fuzzy sets to represent linguistic terms, aiding interpretation across different contexts [22]. These variables encompass definitions, value ranges, and interpretations, making fuzzy logic applicable in decision-making and control systems. By incorporating fuzzy principles, systems can manage uncertainty effectively, offering a framework for reasoning and decision support [23]. Fuzzification is the process of converting precise values into fuzzy values, defining degrees of membership [24]. Fuzzy systems map crisp inputs into crisp outputs using fuzzy rules, involving steps that transform clear data into fuzzy data and vice versa [25].

The next step is evaluating fuzzy rules, which apply fuzzy operators like AND or OR to generate a single result from multiple fuzzy antecedents [26]. Aggregation combines outputs from all rules into a unified fuzzy set, implemented using fuzzy set operations like union or intersection. Finally, defuzzification converts the fuzzy output into a crisp value, often using the centroid defuzzifier, which divides the aggregated set into two equal parts [27]. Membership functions characterize fuzziness, illustrating the degree of truth within fuzzy sets, whether discrete or continuous [24]. They serve as visual tools, guiding problem-solving through experience-based techniques [26]. Key properties include core (complete membership), support (non-zero membership), and boundary (incomplete membership), providing a structured approach to analyze and manage fuzziness.

## 2.3 Investment Strategy via Modern Portfolio Theory

Markowitz's MPT emphasizes efficient resource allocation in financial markets, directing savings towards production investments. Financial markets distribute risk and offer insights into potential returns, guiding investor decisions [28]. Introduced in 1952, MPT aims to maximize expected returns at a given risk level through portfolio selection [29]. It consists of two steps: gathering information and forming expectations about asset performance, followed by selecting a portfolio based on these

expectations. Its important part is diversification, as it reduces individual asset-specific risk, enhancing portfolio stability and minimizing variance [29].

Analytically, MPT defines the optimal portfolio as one that combines multiple assets to maximize expected returns and minimize variance. It applies to various investments, including stocks and bonds, providing a systematic, statistically driven approach to managing portfolios. However, MPT does not account for broader factors affecting investment performance, such as economic changes or market crises, which limit its applicability. MPT relies on several assumptions, including investor rationality, access to all relevant information, and perfectly efficient markets [29]. It also presumes that investors accept higher risks only when compensated by higher returns [19; 30]. Despite criticisms regarding its assumptions, MPT remains valuable for optimizing investments by balancing risk and return.

Financial risk, defined as deviations from expected returns, can be divided into systematic and unsystematic components. Systematic risk affects broad markets and cannot be fully eliminated, while unsystematic risk, specific to individual assets, can be reduced through diversification [30]. The "risk-return" relationship is fundamental to MPT, implying that higher risks require higher expected returns. This is measured using standard deviation, which reflects investment volatility [31]. This study uses MPT to identify the optimal investment portfolio for key areas of AI adoption in the public sector, balancing returns and risks tailored to each country's AI initiatives. Through diversification, MPT ensures a more stable portfolio by distributing investments across different assets, minimizing overall risk while maintaining expected returns [32].

### **3. Case Study**

This study investigates the efficiency of AI implementation within the public sector across various countries. It aims to identify optimal strategies for improving this implementation while considering associated risks. The data for this analysis was sourced from the Oxford AI Readiness Index [1] for the period from 2020 until 2023, which evaluates national AI preparedness across multiple dimensions. Overall Score provided to each country is structured around three key pillars: Government, Technology Sector, and Data and Infrastructure, which are further divided into nine dimensions: Vision, Governance and Ethics, Digital Capacity, Adaptability, Innovation Capacity, Human Capital, Infrastructure, Data Availability, and Data Representativeness. While the original datasets included 10 dimensions, the inconsistent use of "Size" (2020 and 2021) and "Maturity" (2022 and 2023) led to their exclusion to ensure consistency and validity.

A total of 151 countries were included, selected based on the completeness and consistency of their data across all relevant indicators, ensuring reliable analysis. To reduce potential discrepancies caused by variations over time, the study primarily uses average values across the four-year span, providing a stable analytical foundation. However, all four datasets were used during the DEA Window analysis. A synthetic variable with a constant value of 1 was added for all countries to meet the DEA requirement of including both inputs and outputs, as the original dataset provided only output scores. This adjustment allowed for the implementation of output-oriented DEA models, which aim to maximize output for a given level of inputs.

#### *3.1 Efficiency Evaluation Results*

Before conducting the DEA analysis, the data needed to be appropriately prepared, following specific rules to ensure accuracy. To ensure the reliability of the [33] countries with missing values were excluded and the adjustment of zero values to 0.00001 to avoid computational issues [34]. The

dataset adhered to the non-negativity requirement of the CCR model, [35] with all values ranging between 0 and 100. Additionally, the ratio of DMUs (151 countries) to the number of inputs and outputs (9 indicators) satisfied the recommended minimum ratio of 3:1 for adequate discrimination [34]. Indicators were selected to represent key aspects of AI implementation performance [36]. The AI Readiness Index was used to provide a holistic assessment of performance. Even though outliers were detected in adaptability, innovation capacity, and data representativeness for high-performing countries like Singapore and the U.S., they were not removed. Typically, outliers are recommended for removal [37], but these countries were retained in the analysis due to their normalized scores and consistently high performance, as indicated by Oxford's dataset.

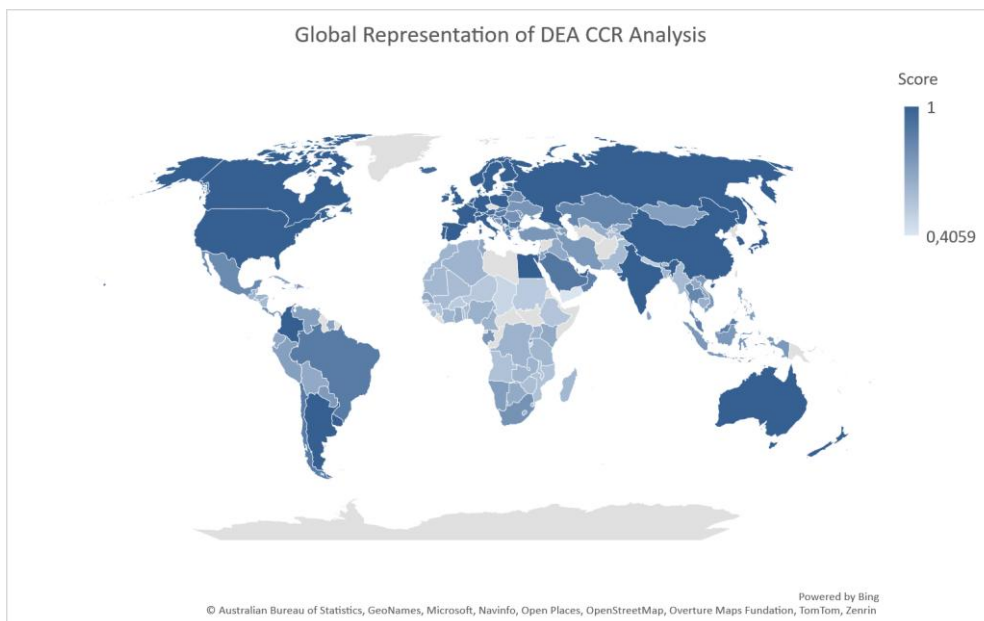
### 3.1.1 Charnes, Cooper, and Rhodes Model

The DEA CCR analysis reveals insights into the efficiency of AI readiness across countries. Top performers like the United States, Singapore, Finland, and the United Kingdom achieved perfect scores of 1, indicating optimal efficiency in using resources to implement AI. This was shown in Table 1:

**Table 1**  
 DMUs identified as efficient based on the CCR analysis

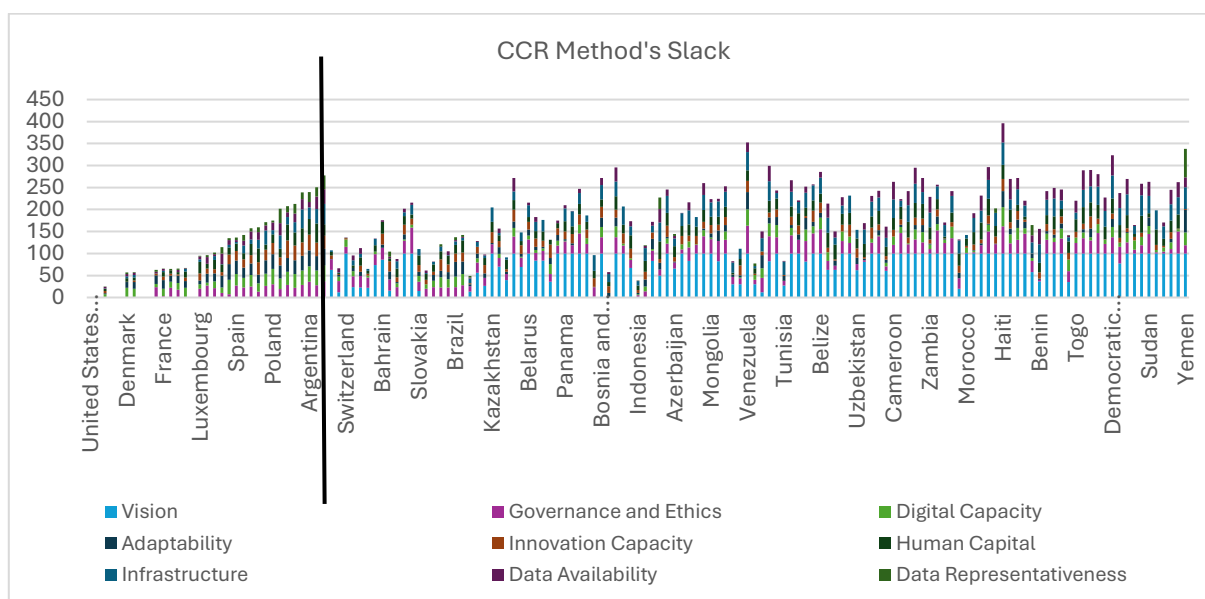
No.	DMU	Score	Rank	No.	DMU	Score	Rank
1	United States of America	1	1	21	Estonia	1	1
2	Singapore	1	1	22	China	1	1
3	Finland	1	1	27	Italy	1	1
4	United Kingdom	1	1	28	Spain	1	1
5	Republic of Korea	1	1	29	Portugal	1	1
7	Denmark	1	1	32	Lithuania	1	1
8	Canada	1	1	33	Malta	1	1
9	Australia	1	1	34	Russian Federation	1	1
10	Japan	1	1	36	Poland	1	1
11	Sweden	1	1	45	India	1	1
12	France	1	1	49	Uruquay	1	1
13	Netherlands	1	1	51	Colombia	1	1
14	Germany	1	1	61	Serbia	1	1
15	Norway	1	1	62	Argentina	1	1
16	Israel	1	1	63	Mauritius	1	1
19	Luxembourg	1	1	77	Egypt	1	1
20	United Arab Emirates	1	1				

In contrast, countries such as Yemen (0.41), Burundi (0.46), and Haiti (0.61) ranked lowest, demonstrating significant inefficiencies. The graph below (Figure 2) shows a graphical representation of the efficiency of states after using the CCR method. States with higher efficiency scores are colored darker, while states in gray are not included in the analysis because data were not available.



**Fig. 2.** Visual representation of the efficiency of states through the CCR method

The results show that key areas such as Vision and Infrastructure have the highest maximum values, while Data Representativeness averages 70.72 out of 100, indicating strong management across nations. In contrast, Human Capital averages 39.54, highlighting a need for capacity building. High slack values in various areas highlight underused resources that could improve outcomes if reallocated. Notably, even efficient countries show minor slacks, such as in Governance and Ethics, suggesting further optimization potential. The slack-based graph below (Figure 3) illustrates efficiency dynamics, with a black line separating efficient countries on the left from inefficient ones on the right. Among the efficient nations, such as Egypt, notable slack exists in dimensions like Infrastructure and Data Representativeness. In contrast, slightly inefficient countries like Ireland demonstrate more balanced resource utilization with minimal slack.

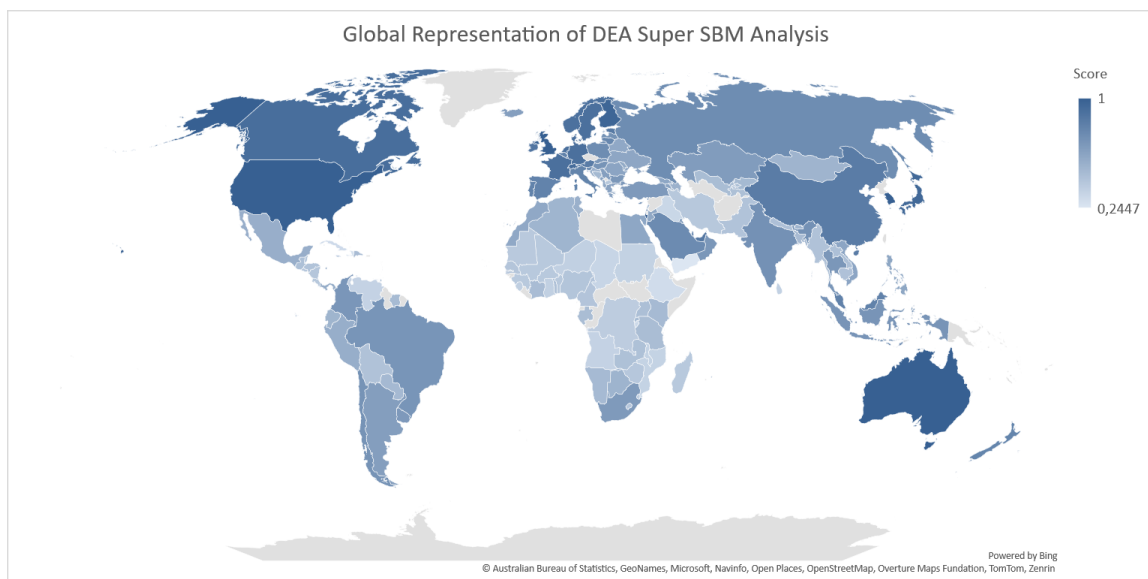


**Fig. 3.** CCR Analysis - Slack

Countries with efficiency scores below 1 display varied slack: Austria and Estonia, for example, show relatively low slack, indicating minor misallocations, while Yemen and Burkina Faso exhibit high slack across most dimensions, signaling fundamental issues in infrastructure, innovation, and human capital. This analysis reveals that achieving full DEA efficiency does not guarantee optimal resource usage. Efficient countries, such as Egypt and Singapore, may still have untapped potential, while some inefficient nations could gain quickly by addressing specific slack areas. Thus, reducing slack is essential for enhancing AI readiness and overall effectiveness, beyond merely achieving a perfect efficiency score.

### 3.1.2 Super Slack-Based Measure Analysis

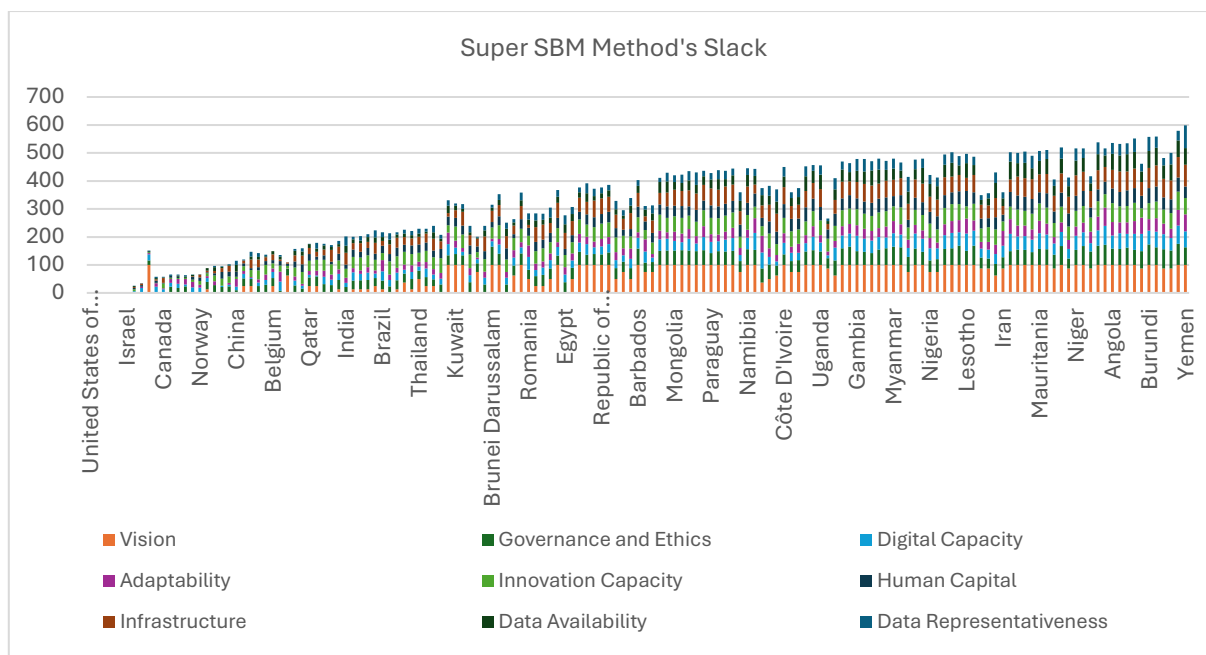
After applying the CCR method, 21.8% of units were classified as efficient. While a high percentage of efficient units is not inherently problematic, it complicates result interpretation, making it hard to distinguish between truly efficient units and those slightly less so. To refine the ranking and better understand unit efficiency, an output-oriented Super-SBM analysis was conducted in the research. This approach reduced the number of efficient units to six, representing 3.97% of the total. Figure 4. reflects this shift, showing fewer darker states, indicating a more accurate ranking of efficiency.



**Fig. 4.** Visual representation of the efficiency of states through the Super SBM method

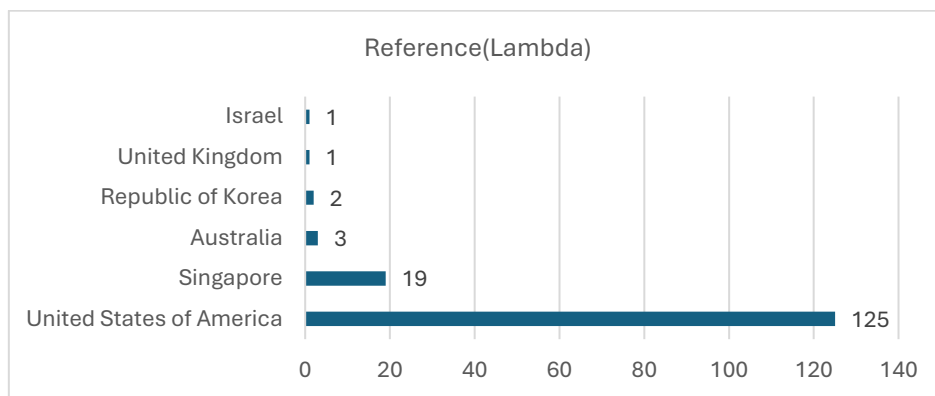
In the DEA Super SBM analysis, the United States, Singapore, Australia, South Korea, Israel and the United Kingdom achieve the highest super-efficiency scores, while Haiti and Yemen show the lowest, highlighting inefficiencies. High scores in Data Representativeness and Availability contrast with weaker areas like Innovation Capacity and Human Capital. Efficient countries have balanced resource allocation, whereas inefficient countries show imbalances, particularly in Human Capital.

Unlike the CCR analysis, countries with a Super-SBM efficiency score of 1 exhibit no slack, indicating optimal input usage. The slack graph (Figure 5) shows that fully efficient countries like the United States, Singapore, and South Korea have no slack, while nations like Ethiopia, Iraq, and Angola reveal substantial slack, indicating underused resources.



**Fig. 5.** Super SBM Analysis - Slack

Since six countries received a score of 1 and shared the first rank based on the DEA analysis, a total ranking was manually created by considering the number of reference values. The resulting ranking places the United States in the first position, followed by Singapore, Australia, South Korea, and a shared rank between Israel and the United Kingdom. The number of times that each country is used as a reference is represented in the graph below (Figure 6).



**Fig. 6.** Distribution of Lambda Reference Values Among Top-Ranked Countries

### 3.1.3 Window Analysis

Given that the analysis thus far has relied solely on average values across the four-year period (2020–2023), it was essential to conduct a more granular evaluation that examines each period independently. To achieve this, Window analysis was employed on the dataset.

Figure 7 presents the statistical outcomes of the DEA Window analysis, detailing the maximum, minimum, average, and standard deviation (SD) values obtained throughout the analysis. Vision scores reach 100, with a broad SD (41–47) indicating global disparity. Governance, Digital Capacity, and Innovation Capacity averages improve slightly, while Adaptability and Human Capital fluctuate with high SDs. Infrastructure and Data Availability show upward trends but retain low minimums, highlighting gaps.

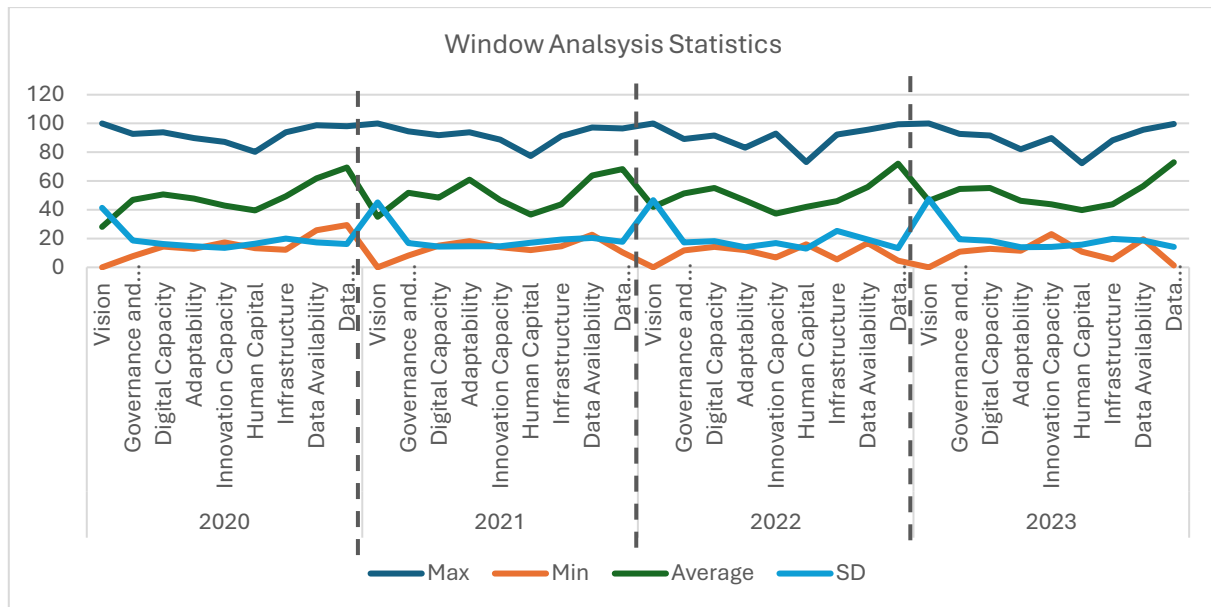


Fig. 7. Statistics on Input/Output from Window analysis

Countries like Argentina, Australia, Canada, China, and the United States consistently maintain perfect efficiency scores, showing optimized resource use. Interestingly, Israel does not maintain its efficiency in the DEA Window analysis, despite being classified as efficient in both the CCR and Super-SBM models. This discrepancy is due to the dynamic nature of the Window analysis, which evaluates efficiency across multiple overlapping time periods rather than relying solely on average values. While Israel demonstrated overall efficiency in static models, the Window analysis highlights its performance inconsistencies over time. In the Table 2 below are shown DMUs that are efficient, and inefficient DMUs up to Israel.

**Table 2**  
 Efficient and some inefficient DMUs based on Window analysis

DMUs	Average	C-Average	DMUs	Average	C-Average
Argentina	1	1	Australia	1	1
Canada	1	1	China	1	1
Colombia	1	1	Denmark	1	1
Egypt	1	1	Estonia	1	1
Finland	1	1	France	1	1
Germany	1	1	India	1	1
Italy	1	1	Japan	1	1
Lithuania	1	1	Luxembourg	1	1
Malta	1	1	Mauritius	1	1
Netherlands	1	1	Norway	1	1
Poland	1	1	Portugal	1	1
Republic of Korea	1	1	Russian Federation	1	1
Serbia	1	1	Singapore	1	1
Spain	1	1	Sweden	1	1
United Arab Emirates	1	1	United Kingdom	1	1
United States of America	1	1	Uruguay	1	1
Switzerland	0.992122	0.992121958	Ireland	0.985273	0.985273
Belgium	0.9827333	0.982733328	Qatar	0.976432	0.976432
Latvia	0.9749051	0.974905139	Israel	0.973012	0.973012

Some countries show variable trends: Bangladesh, Paraguay, and Nigeria improved efficiency in 2022–2023, while Armenia and Benin saw slight declines, reflecting stability challenges. Brazil and Hungary achieved perfect efficiency recently despite earlier lower scores, indicating progress. Yemen, Sudan, and Angola consistently showed lower efficiency, highlighting the need for input-output optimization.

### 3.2 Risk Analysis Findings

The Fuzzy Logic was analyzed through MATLAB which assessed risk levels for countries aiming to improve public sector AI efficiency, enabling informed decisions on resource allocation and portfolio management. Fuzzy Logic rules were established using Microsoft Excel to analyze risk levels from different investment scenarios, with risk values ranging between 181 and 243. This range facilitated adjustments during the test portfolio analysis using Excel’s Solver and was consequently applied in this step. To create a manageable analysis, the research focused on three primary pillars—Government, Technology Sector, and Data & Infrastructure—due to the complexity of evaluating all nine dimensions, which would have generated 262,144 rules. As the selected pillars represent average values of key dimensions, it is allowing a simplified but accurate analysis.

Before creating the rules, the input and output linguistic variables and their membership functions were defined. These functions must clearly describe the relevant characteristics that will be the subject of analysis [38]. Membership functions were divided into five categories: Very Small (VS), Small (S), Medium (M), Large (L), and Very Large (VL), corresponding to the risk categories. A total of 64 rules were generated, classified into the five risk groups mentioned above. The defined functions were carefully structured to ensure consistency and avoid contradictions, thus maintaining the system’s functionality. Rules were aligned to ensure reliable risk management and precise analysis [39]. The approach focused on increasing investments in specific pillars to reduce portfolio risk, establishing that lower investment percentages correlate with higher risk. This method improved risk management and informed investment decisions by balancing investments and acceptable risk levels. Rule weights were determined by three criteria:

- i. Ensuring that the sum of membership function weights for S in the pillars was less than VL for risk.
- ii. Aligning the sum of VL membership function weights for the pillars with VS for risk.
- iii. Achieving a normal distribution of weights, following a Gaussian curve.

Weights were assigned to the risk categories, starting with 0.2 for VL and incrementing by 0.2 for each subsequent category, reaching 1 for VS. Each pillar was assumed to have equal importance, resulting in evenly distributed weights, as shown in Figure 8.

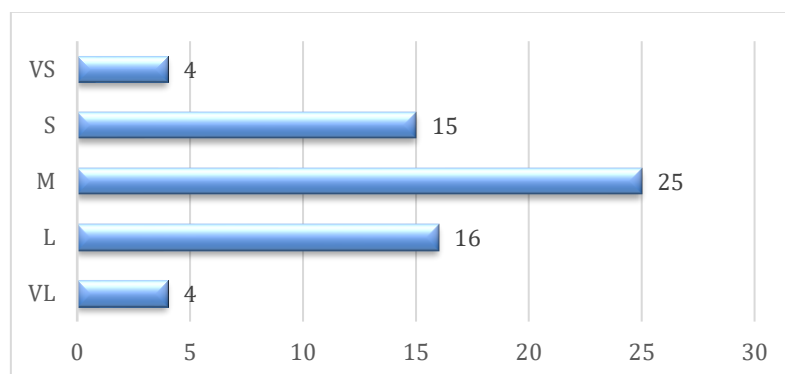


Fig. 8. Number of rules per function and their distribution

After implementing the rules in MATLAB, the resulting structure of membership functions for the three pillars was visualized (Figures 9 and 10). The graphical representation showed the membership distribution across categories, with different colors representing the levels from small to very large. This clear depiction of the range allowed for the inclusion of all relevant values from the dataset.

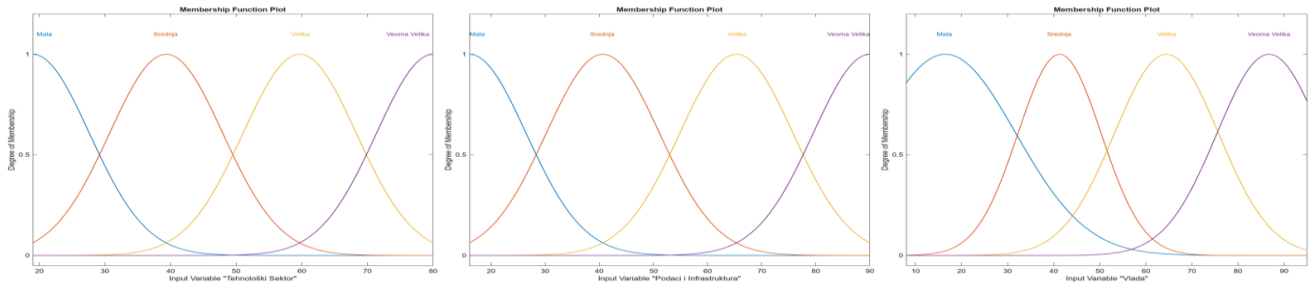


Fig. 9. Representation of membership functions for pillars

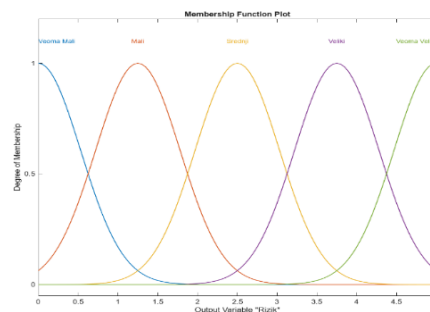


Fig. 10. Representation of risk function for pillars

A 3D plot (Figure 11.) was generated, demonstrating the relationship between two input variables (any two pillars) and the output variable (risk). The analysis revealed the risk levels for each country over the observed period. The results, presented in Table 3, show the 20 countries with the lowest risk levels and the 10 with the highest.

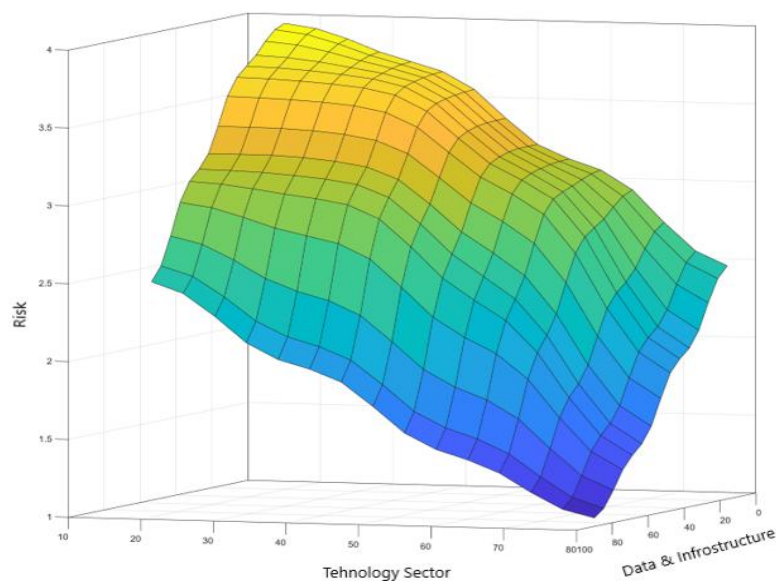


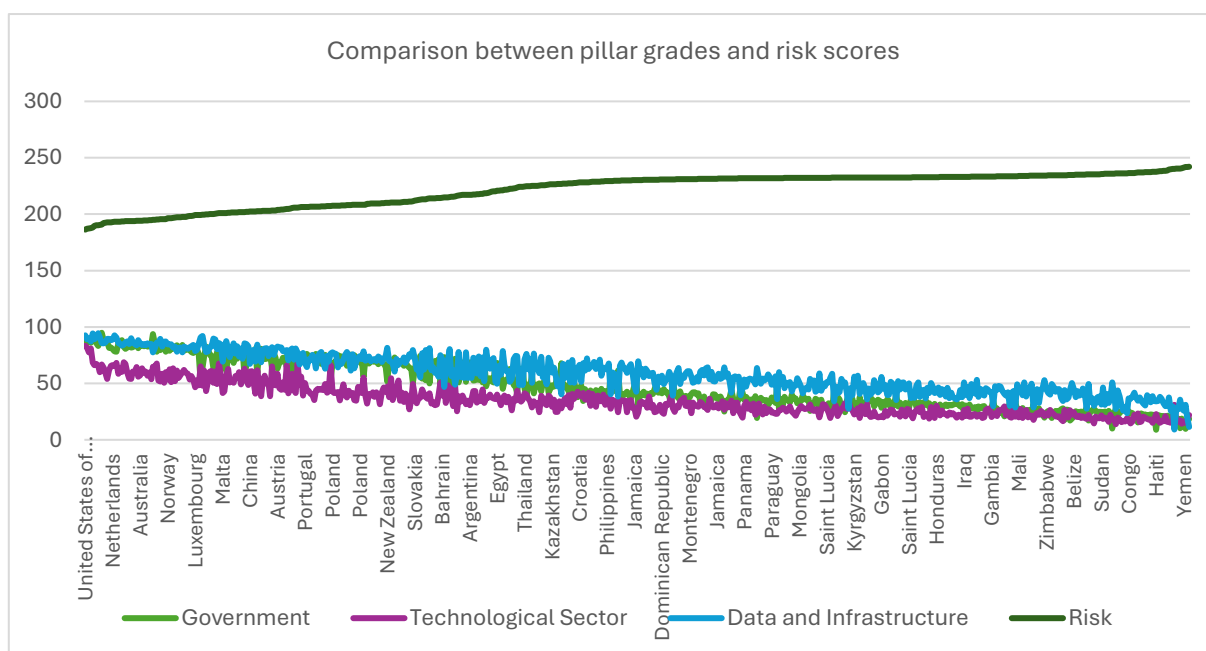
Fig. 11. 3D representation of the rules

The United States consistently maintained the lowest risk levels across the four-year period, followed by Singapore, the United Kingdom, and Finland, as shown in Table 3. This reflects their high efficiency in AI implementation, as confirmed by their top efficiency scores in the DEA analysis. Conversely, countries with lower AI efficiency scores, such as Yemen, Haiti, and the Democratic Republic of Congo, exhibited the highest risk levels, mirroring the results of the DEA analysis. This correlation suggests that countries' risk levels are directly tied to their AI efficiency scores, highlighting the importance of strategic resource allocation to improve AI adoption in the public sector.

**Table 3**  
 Risk that first 10 and last 5 countries face

Year	Country	Risk	Normalized Risk
2021	United States	186.376	0.000
2022	United States	187.106	0.013
2020	United States	187.413	0.019
2023	United States	187.548	0.021
2022	Singapore	188.193	0.033
2023	Singapore	189.676	0.059
2021	United Kingdom	190.135	0.068
2020	United Kingdom	190.319	0.071
2020	Finland	190.548	0.075
2021	Singapore	190.650	0.077
2022	Democratic Republic of Congo	240.745	0.977
2020	Yemen	241.413	0.989
2022	Haiti	241.882	0.997
2021	Yemen	241.896	0.998
2022	Yemen	242.030	1.000

Figure 12. shows a reverse correlation between the score's countries received for their pillars and the associated risk; higher scores correspond to lower risk scores as determined by fuzzy logic in MATLAB.



**Fig. 12.** Comparison between pillar grades and risk scores

### 3.3 Optimal Portfolio Design

The final step involved creating an optimal investment portfolio using Excel based on Markowitz's Modern Portfolio Theory. The goal was to achieve the highest possible return within a given risk level. Unlike traditional stock price data, this study applied MPT to determine an optimal AI investment strategy across different countries by using efficiency and risk scores derived from previous analyses.

Initially, the Excel file was structured to accept percentage changes in stock values over time (as shown in Table 4). For this analysis, the format was adapted to include data for countries over four years, categorized by dimensions (shown in Table 5). This adaptation enabled calculation of optimal portfolios for AI implementation, considering both efficiency and risk factors established earlier.

**Table 4**

Initial view of the Excel table, created based on Markowitz's theory, used for creating an optimal trading portfolio

Date of Trading	LSTA	KMBN	JMBN	JESV	FITO	ENHL	DNOS	DINNPB
30.12.2016	22.77	2.18	2.28	1.81	0.04	0.7	0	0
29.12.2016	-19.2	0.06	0	-6.35	0	1.36	0	0
28.12.2016	0	-1.68	0	0	0	0.14	0	-4.9
27.12.2016	0	2.79	0	-2.27	0	-0.14	0	0.51
26.12.2016	0	0	0	0	0	0.07	0	0.41
23.12.2016	0	-1.12	0	0.64	-0.66	-2.84	-2.07	-1.31
22.12.2016	0	-2.13	0	-0.64	0.62	0.07	0	-0.18
21.12.2016	0	0	0	1.29	-1.09	2.56	0	0
20.12.2016	0	0.17	0	0.56	0	-1.06	0	0
Date of Trading	AERO	TGAS	SJPT	PRGS	NIIS	MTLC	KOPB	
30.12.2016	0.52	-2.91	0	-11.11	-0.13	0	0	
29.12.2016	-1.54	0	-0.32	0	0.68	0.36	1.59	
28.12.2016	0.34	0	-3.12	0	0	0	0	
27.12.2016	0.78	4	0	5.88	-0.14	1.57	0	
26.12.2016	-0.09	-2.49	0	0	0	-1.49	1.29	
23.12.2016	-2.12	-1.79	0	0	0.14	0	0	
22.12.2016	0.6	0	0	0	-0.41	-0.3	0	
21.12.2016	1.56	0	0	0	0	2.56	0	
20.12.2016	0.09	0	0	0	-0.4	0	-2.51	

**Table 5**

The Excel table, created based on Markowitz's theory, when applied to data from the AI Readiness Index

Country	Vision	Governance and Ethics	Digital Capacity	Adaptability	Innovation Capacity
Albania 20	0	52.22	62.43	47.51	32.2
Algeria 20	0	40.03	31.3	44.69	33.09
Angola 20	0	31.19	41.11	30.21	25.81
Argentina 20	100	45.67	51.69	46.22	37.32
Armenia 20	0	49.91	56.21	50.1	43.58
Australia 20	100	86.44	64.56	67.46	61.99
Austria 20	50	59.48	66.23	68.5	66.29
Azerbaijan 20	0	56.34	70.88	61.71	51.81
Bahrain 20	0	55.26	72.37	61.96	50.18

**Table 5**  
 Continued

Country	Human Capital	Infrastructure	Data Availability	Data Representativeness
Albania 20	29.58	42.19	68.7	56.75
Algeria 20	35.95	37.92	51.66	51.89
Angola 20	16.29	27.03	31.1	32.03
Argentina 20	35.21	44.29	71.76	66.24
Armenia 20	34.49	45.26	72.27	59.27
Australia 20	65.69	83.95	90.15	93.97
Austria 20	65.27	84.85	87.08	90.44
Azerbaijan 20	37.97	48.7	71.22	61.33
Bahrain 20	33.36	82.27	74.35	82.54

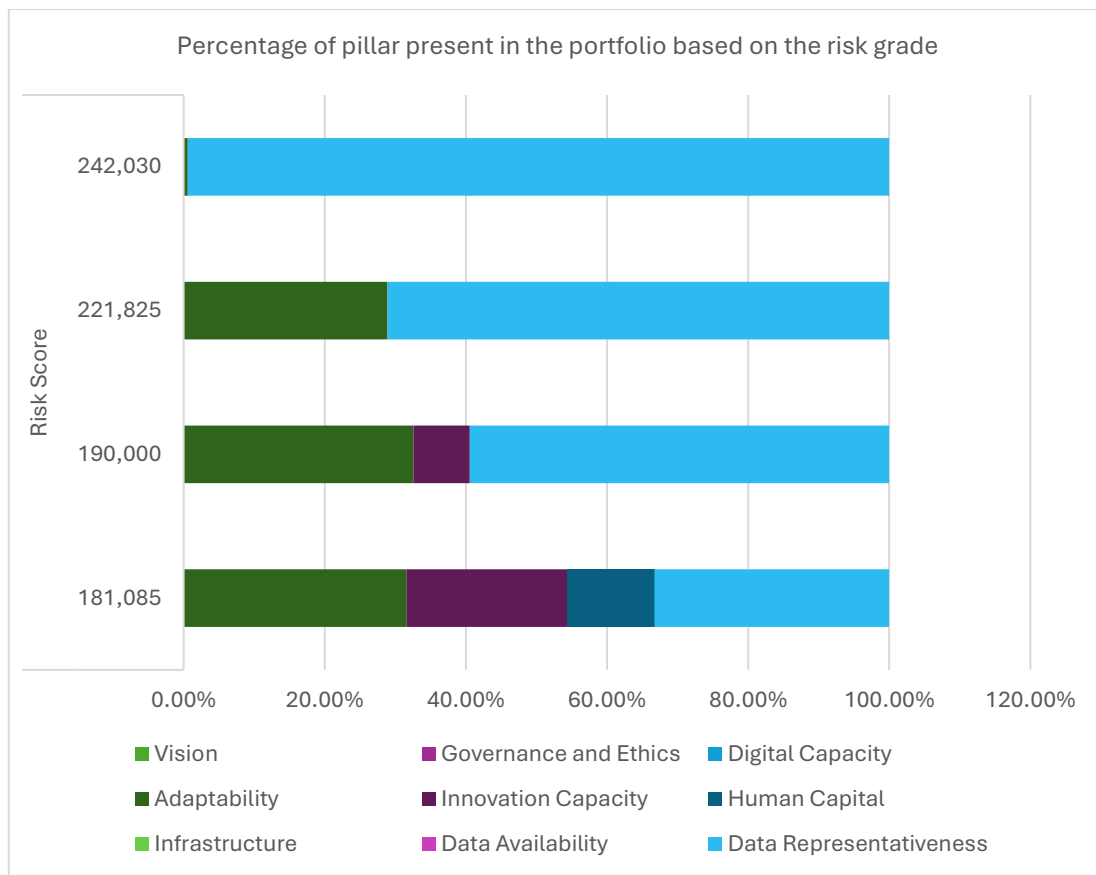
After inputting the efficiency and risk values, ranging from 181 to 243, the following investment percentages across dimensions were calculated for varying risk levels (Table 6).

**Table 6**  
 Distribution of Dimensions in Portfolio under Specific Risk Levels

Risk Level	181.085	190.000	221.825	242.030
Vision	0.00%	0.00%	0.00%	0.00%
Governance & Ethics	0.00%	0.00%	0.00%	0.00%
Digital Capacity	0.00%	0.00%	0.00%	0.00%
Adaptability	31.60%	32.56%	28.91%	0.60%
Innovation Capacity	22.75%	8.00%	0.00%	0.00%
Human Capital	12.35%	0.00%	0.00%	0.00%
Infrastructure	0.00%	0.00%	0.00%	0.00%
Data Availability	0.00%	0.00%	0.00%	0.00%
Data Representativeness	33.29%	59.44%	71.09%	99.40%

The analysis reveals a clear tendency toward reduced diversification with increasing risk levels (Figure 13). As risk increases, investment focuses on fewer, more reliable dimensions, aiming to minimize potential losses. The risk value of 221.825 represents the average global risk score, suggesting that countries should consider this as a benchmark for AI investments. It can serve as a reference point for aligning national strategies with global standards. Data Representativeness and Adaptability are consistently present across all portfolios, albeit with varying emphasis. The strong link between these dimensions suggests their importance in successful AI implementation in the public sector. As risk levels rise, the proportion of Data Representativeness increases significantly, underscoring its critical role. Accurate data representation enables informed decision-making, leading to better AI outcomes. Adaptability, on the other hand, ensures timely responses to changing technological and policy environments, vital for maintaining public sector efficiency.

These findings align with Oxford’s report indicating that regions with low average scores in Data Availability and Data Representativeness could face challenges in future AI applications [1]. The divergence between the recommendation that Oxford provided and results obtained in this research, as 0% was given to Data Availability, could be due to the fact that the maximal Data Availability was falling over the years, as shown by DEA Window analysis.



**Fig. 13.** Percentage of pillar present in the portfolio based on the risk grade

The results indicate that, for optimal AI outcomes, countries should focus on investments in Data Representativeness and Adaptability rather than Data Availability. The balance between risk management and strategic investment can enhance AI readiness while maintaining agility in the public sector. Adjusting investments according to risk levels enables countries to refine strategies, achieve sustained AI development, and effectively manage uncertainties.

#### 4. Discussion

The proposed model, integrating DEA, Fuzzy Logic, and MPT, offers a novel approach to evaluating AI implementation in the public sector.

Unlike traditional models that separately address efficiency measurement or risk assessment, this model unifies these elements into a single framework. Traditional approaches often rely on static DEA for single-period efficiency assessments, which fail to capture temporal variations or evolving public sector conditions. The proposed model addresses this limitation by incorporating DEA Window Analysis, enabling a dynamic evaluation of efficiency trends over time and providing insights that align with changing conditions. By integrating Fuzzy Logic, the model accounts for uncertainties in AI adoption, complementing efficiency evaluation with strong risk assessment. The inclusion of MPT enhances the framework by providing specific guidance on resource allocation, tailored to each country's efficiency and risk profile. This comprehensive approach simultaneously evaluates efficiency, manages risk, and informs investment strategies, distinguishing it from existing methodologies.

Conventional models often assume clear efficiency boundaries and overlook the uncertainties inherent in AI adoption. They may offer general investment suggestions but rarely specify how

resources should be distributed across different AI dimensions or countries. The proposed model addresses these gaps through the integration of MPT, optimizing resource allocation and supporting strategic decision-making. Additionally, its ability to dynamically assess efficiency trends and incorporate risk factors ensures that governments can adapt AI strategies to new data or evolving trends. By uniting DEA, Fuzzy Logic, and MPT, the proposed model not only evaluates current performance but also provides actionable recommendations for resource distribution. This approach supports informed policy planning, minimizes risks, and maximizes the benefits of AI in public sector transformation, setting it apart from static, single-dimensional methods.

## **5. Conclusion**

This study provides a evaluation of AI implementation in the public sector across various countries, focusing on efficiency, risk, and optimal resource allocation. Using a combination of DEA, Fuzzy Logic, and MPT, the research identifies top-performing countries in AI adoption, such as the United States, Singapore, and Finland, while highlighting inefficiencies in countries like Yemen, Burundi, and Haiti. The integration of DEA Window Analysis reveals temporal trends, demonstrating dynamic changes in AI readiness over time. Fuzzy Logic offers insights into risk levels associated with AI adoption, identifying key factors like Data Representativeness and Adaptability as crucial for risk reduction. MPT aids in determining the optimal distribution of investments, maximizing efficiency while minimizing risks.

The methodology used in this study is distinctive for its simultaneous evaluation of efficiency, risk, and investment strategy. Unlike traditional approaches, which often focus on a single aspect, this multi-dimensional framework provides a holistic analysis that adapts to evolving data and changing conditions in the public sector. The proposed approach offers more realistic assessments and practical guidance for AI implementation, helping in the improvement of policy planning and strategic decision-making.

However, certain limitations were identified during the analysis, primarily related to data availability and scope. The model's reliance on available data may lead to potential biases, as not all countries provide consistent and high-quality data. Additionally, while part of this study (risk calculation) focused on three core pillars for simplicity, future research should consider incorporating additional dimensions to achieve a more detailed evaluation, especially since one dimension was excluded from this research. Expanding the model to include more dynamic and real-time data sources could improve its predictive capabilities and offer more precise policy recommendations. Furthermore, exploring sector-specific AI implementations could offer more tailored insights, providing clearer guidance for resource allocation within specific public sectors.

To further develop the model, it would be beneficial to incorporate an additional methodology capable of projecting potential improvements in efficiency if countries were to implement the proposed portfolios under their specific risk levels. This predictive component would provide clearer estimates of expected outcomes, enabling more informed decision-making and strategic planning for AI adoption.

## **Acknowledgement**

This research was not funded by any grant.

## **Conflicts of Interest**

The authors declare no conflicts of interest.

## References

- [1] Hankins, E., Nettel, P. F., Martinescu, L., Grau, G., & Rahim, S. (2023). GovernmentAI Readiness Index 2023. Oxford Insights.
- [2] Bhima, B., Az Zahra, A. R., & Nurtino, T. (2023). Enhancing Organizational Efficiency through the Integration of Artificial Intelligence in Management Information Systems. *APTISI Transactions on Management (ATM)*. <https://doi.org/10.33050/atm.v7i3.2146>
- [3] Keppeler, F. (2023). No Thanks, Dear AI! Understanding the Effects of Disclosure and Deployment of Artificial Intelligence in Public Sector Recruitment. *SSRN Electronic Journal*. <https://doi.org/10.2139/ssrn.4232641>
- [4] Li, J. (2022). Application of Improved DEA Algorithm in Public Management Problem Classification. *Computational Intelligence and Neuroscience*. <https://doi.org/10.1155/2022/2568162>
- [5] Krejnus, M., Stofkova, J., Stofkova, K. R., & Binasova, V. (2023). The Use of the DEA Method for Measuring the Efficiency of Electronic Public Administration as Part of the Digitization of the Economy and Society. *Applied Sciences*. <https://doi.org/10.3390/app13063672>
- [6] Dahmardeh, N., & Pourshahabi, V. (2011). Agility evaluation in public sector using fuzzy logic. *Iranian Journal of Fuzzy Systems*. <https://doi.org/10.22111/ijfs.2011.289>
- [7] Jana, C., Senapati, T., & Pal, M. (2019). Handbook of research on emerging applications of fuzzy algebraic structures. IGI Global.
- [8] Rasoulzadeh, M., Edalatpanah, S. A., Fallah, M., & Najafi, S. (2022). A multi-objective approach based on Markowitz and DEA cross-efficiency models for the intuitionistic fuzzy portfolio selection problem. *Decision Making: Applications in Management and Engineering*. <https://doi.org/10.31181/dmame0324062022e>
- [9] Mashayekhi, Z., & Omrani, H. (2016). An integrated multi-objective Markowitz-DEA cross-efficiency model with fuzzy returns for portfolio selection problem. *Applied Software Computing*. <https://doi.org/10.1016/j.asoc.2015.09.018>
- [10] Boussofiane, A., Dyson, R., & Thanassoulis, E. (1991). Applied data envelopment analysis. *European Journal of Operational Research*. [https://doi.org/10.1016/0377-2217\(91\)90331-O](https://doi.org/10.1016/0377-2217(91)90331-O)
- [11] Cook, W., & Seiford, L. (2009). Data envelopment analysis - Thirty years on. *European Journal of Operational Research*, 1-17. <https://doi.org/10.1016/j.ejor.2008.01.032>
- [12] Amirteimoori, A., Zadmirzaei, M., & Hassanzadeh, F. (2021). Developing a new integrated artificial immune system and fuzzy non-discretionary DEA approach. *Soft Computing*. <https://doi.org/10.1007/s00500-021-05725-1>
- [13] Cooper, W., Seiford, L., & Tone, K. (2005). *Introduction to Data Envelopment Analysis and Its Uses*. Springer. <https://doi.org/10.1007/0-387-29122-9>
- [14] Popović, M., Savić, G., Kuzmanović, M., & Martić, M. (2020). Using data envelopment analysis and multi-criteria decision-making methods to evaluate teacher performance in higher education. *Symmetry*, 12(4), 563. <https://doi.org/10.3390/sym12040563>
- [15] Tone, K. (2016). On the Consistency of Slacks-based Measure-Max Model and. *National Graduate Institute for Policy Studies*, 1-11. <https://doi.org/10.13189/ujm.2017.050307>
- [16] Sueyoshi, T., Goto, M., & Sugiyama, M. (2013). DEA window analysis for environmental assessment in a dynamic time shift: Performance assessment of U.S. coal-fired power plants. *Energy Economics*, 40, 845-857. <https://doi.org/10.1016/j.eneco.2013.09.020>
- [17] Coelli, T. (1998). A multi-stage methodology for the solution of orientated DEA models. *Operational Research Letters*. [https://doi.org/10.1016/S0167-6377\(98\)00036-4](https://doi.org/10.1016/S0167-6377(98)00036-4)
- [18] Dyson, R., Allen, R., Camanho, A., Podinovski, V., Sarrico, C., & Shale, E. (2001). Pitfalls and protocols in DEA. *European Journal of Operational Research*. [https://doi.org/10.1016/S0377-2217\(00\)00149-1](https://doi.org/10.1016/S0377-2217(00)00149-1)
- [19] Novák, V., Perfilieva, I., & Mockor, J. (1999). Mathematical principle. <https://doi.org/10.1007/978-1-4615-5217-8>
- [20] Zadeh, L. A. (1965). Fuzzy Sets. *Information and Control*, 8(3), 338–353. [https://doi.org/10.1016/S0019-9958\(65\)90241-X](https://doi.org/10.1016/S0019-9958(65)90241-X)
- [21] Yager, R. R., & Filev, D. P. (1994). *Essentials of fuzzy modeling and control*. New York.
- [22] Huynh, V., & Ho, N. C. (2002). An algebraic approach to linguistic hedges in Zadeh's fuzzy logic. *Fuzzy Sets and Systems*, 229-254. [https://doi.org/10.1016/S0165-0114\(01\)00147-6](https://doi.org/10.1016/S0165-0114(01)00147-6)
- [23] Paudel, G., Pahari, N., & Kumar, S. (2022). Application of Fuzzy Logic Through Bellmen-Zadeh Maximin Method. *Journal of Nepal Mathematical Society*. <https://doi.org/10.3126/jnms.v5i1.47376>
- [24] Ross, T. (2000). *Membership Functions, Fuzzification and Defuzzification*. Springer. [https://doi.org/10.1007/978-3-7908-1859-8\\_3](https://doi.org/10.1007/978-3-7908-1859-8_3)
- [25] Mendel, J. (1995). Fuzzy logic systems for engineering: a tutorial. *Proceedings of the IEEE*, 83(3), 345-377. <https://doi.org/10.1109/5.364485>

- [26] Caulfield, H. (1996). Fuzzy Optical Metrology. *IEEE Transactions on Fuzzy Systems*. [https://doi.org/10.1016/S0925-2312\(02\)00698-7](https://doi.org/10.1016/S0925-2312(02)00698-7)
- [27] Loan, S., Murshid, A. M., & Bashir, F. (2013). A Novel VLSI Architecture of a Defuzzifier Unit for a Fuzzy Inference Processor. *International Conference on Advanced Electronic Systems*. <https://doi.org/10.1109/ICAES.2013.6659378>
- [28] Markowitz, H., Miller, M., & Sharpe, W. (1990). The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 1990. The Royal Swedish Academy of Sciences.
- [29] Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance*. <https://doi.org/10.1111/j.1540-6261.1952.tb01525.x>
- [30] Mangram, M. E. (2013). A Simplified Perspective of the Markowitz Portfolio Theory. *Global Journal of Business Research*, 7(1), 59-70.
- [31] Bradford, J., & Miller, T. (2008). *Fundamentals of investments* (5th ed.). McGraw-Hill.
- [32] Fabozzi, F., Gupta, F., & Markowitz, H. (2002). The Legacy of Modern Portfolio Theory. *Institutional Investor, Inc.* <https://doi.org/10.3905/joi.2002.319510>
- [33] Sarkis, J. (2002). Preparing Your Data for DEA. *Productivity Analysis in the Service Sector with Data Envelopment Analysis* (2nd ed.). [https://doi.org/10.1007/978-0-387-71607-7\\_17](https://doi.org/10.1007/978-0-387-71607-7_17)
- [34] Bowlin, W. (1998). Measuring Performance: An Introduction to Data Envelopment Analysis (DEA). *Journal of Cost Analysis*, 7, 3-27. <https://doi.org/10.1080/08823871.1998.10462318>
- [35] Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *Journal of Productivity Analysis*, 2, 197–237. [https://doi.org/10.1016/0377-2217\(78\)90138-8](https://doi.org/10.1016/0377-2217(78)90138-8)
- [36] Mingers, J., Liu, W., & Meng, W. (2009). Using SSM to structure the identification of inputs and outputs in DEA. *Journal of the Operational Research Society*. <https://doi.org/10.1057/palgrave.jors.2602542>
- [37] Yang, M.-Y., Wan, G., & Zheng, E. (2014). A predictive DEA model for outlier detection. *Journal of Management Analytics*. <https://doi.org/10.1080/23270012.2014.889911>
- [38] Olesiak, K. (2017). Application of Fuzzy Logic Toolbox for Modelling Fuzzy Logic Controllers. *SOCIETY. INTEGRATION. EDUCATION. Proceedings of the International Scientific Conference*. <https://doi.org/10.17770/sie2017vol3.2398>
- [39] Castellanos, L. E. (2019). Programming of Job Shop Production Systems with Fuzzy Logic. *SciELO*. <https://doi.org/10.18180/tecciencia.2018.25.5>